Re-exam Calculus-3 (10 points free):

Total points to obtain 100 [*In all problems provide a brief justification for what you do]*

Problem 1 (15 points)

Consider the series: $\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}} x^n$; For which values of x is the series convergent?

Problem 2 (15 points)

Find the value of c if

$$\sum_{n=2}^{\infty} (1 + c)^{-n} = 2$$

Problem 3 (15 points)

If a, b, and c are all positive constants and $y(x)$ is a solution of the differential equation $ay'' + by' + cy = 0$, show that $\lim_{x\to\infty} y(x) = 0$.

Tip : consider all possible cases for $b^2 - 4ac$ $(\le 0,\ > 0)$

Problem 4 (20 points)

(a: 10 points)
(b: 10 points)

Let L be a nonzero real number.

(a) Show that the boundary-value problem $y'' + \lambda y = 0$, $y(0) = 0$, $y(L) = 0$ has only the trivial solution $y = 0$ for the cases $\lambda = 0$ and $\lambda < 0$.

(b) For the case $\lambda > 0$, find the values of λ for which this problem has a nontrivial solution and give the corresponding solution.

Problem 5 (10 points)

Assume a function *f(x)* to have Fourier transform: $F(k) = \int_{-\infty}^{+\infty} f(x) e^{-i2\pi kx} dx$

Consider the Fourier transform expression for the Dirac Delta function: $\delta(k) = \int_{-\infty}^{+\infty} e^{-i2\pi kx} dx$

Derive the Fourier Transform of the function: $f(x) = \cos[2\pi k_o x]$

Problem 6 (15 points)

Find the periodic solutions in complex form of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{ky} = \mathrm{f(x)}$

where k is a constant (real number), and f (x) is a known 2π-periodic function.

Tip: Consider the Fourier series expansions in complex form: $f(x) = \sum_{n=-\infty}^{n=+\infty} f_n e^{inx}, \quad y(x) = \sum_{n=-\infty}^{n=+\infty} y_n e^{inx}$